Section: _____

Instructor: _____

MATH 137: INTERMEDIATE ALGEBRA

FALL 2008 FINAL EXAM – (100 PTS)

NAME: Answer Key Date: _____

You have 2 hours to complete this exam, which is formatted in two sections (A and B). Section A is worth 25 points while Section B is worth 75 points. Please note that graphing calculators, cheat sheets or access to formulae are not permitted throughout this exam. Also, cellular telephones cannot be used as calculators on the exam. Be sure to erase or cross out completely all work that is not to be graded. Moreover, do not offer two or more different answers or solutions to the same problem since doing so will result in loss of all credit for the problem. Do not give decimal approximations for any of the problems on this exam.

SECTION A – (25 PTS): In this section, no partial credit will be awarded. Work carefully and accurately on scrap paper, and then provide only your final answers in simplest form in the indicated spaces.

A1. Simplify the following radical expressions completely. You may assume that the variables are positive.

a). $7\sqrt{45} - 2\sqrt{5}$ **(2 pts)**

Answer (a): $19\sqrt{5}$

$7\sqrt{9}\cdot\sqrt{5} - 2\sqrt{5}$

$7\cdot 3\sqrt{5} - 2\sqrt{5}$

$21\sqrt{5} - 2\sqrt{5}$

b). $\dfrac{2\sqrt{3}}{7\sqrt{5}}$ **(2 pts)**

Answer (b): $\dfrac{2\sqrt{15}}{35}$

$\dfrac{2\sqrt{3}}{7\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}$

$\dfrac{2\sqrt{15}}{35}$

c). $\sqrt[3]{54x^5}$ **(2 pts)**

Answer (c): $3x\sqrt[3]{2x^2}$

$\sqrt[3]{27x^3}\cdot\sqrt[3]{2x^2}$

A2. Simplify the following complex numbers completely.

a). $\sqrt{-48}$ **(2 pts)**

Answer (a): $4i\sqrt{3}$

$\sqrt{16} \cdot \sqrt{-1} \cdot \sqrt{3}$

b). $(5-6i)(2+5i)$ **(2 pts)**

Answer (b): $40+13i$

$10-12i+25i-30i^2$

$10+13i+30$

A3. Perform the following tasks.

a). Identify the vertex of the parabola $y=(x+2)^2-3$ **(2 pts)**

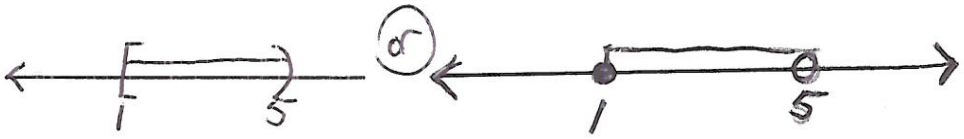
Answer (a): $(-2,-3)$

b). Solve and graph the inequality $-5 \le 2x-7 < 3$. **(2 pts)**

$2 \le 2x < 10$

$1 \le x < 5$

Answer (c): $1 \le x < 5$



c). Find, if any, the value(s) for which $f(x)=\dfrac{x^2-9}{4x-12}$ is undefined. **(2 pts)**

Answer (d): $x \neq 3$

$f(x)$ is undefined when $x=3$

$4(x-3)$

Solution to changed problem should be

Domain = $\{x : x \neq 3\}$

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A4. Consider the parabola $f(x) = x^2 + 4x - 5$

a). Does the parabola open upward or downward? **(2 pts)**

Answer (a): upward

b). Find the vertex of the parabola. **(2 pts)**

axis: $x = \frac{-b}{2a} = \frac{-4}{2} = -2$

$y = (-2)^2 + 4(-2) - 5$

$4 - 8 - 5$

Answer (b): $(-2, -9)$

let it go

y/o ( )

c). Find all x-intercepts and the y-intercept of the parabola.

$0 = (x+5)(x-1)$

Answer for x-intercepts: $(-5, 0), (1, 0)$ **(2 pts)**

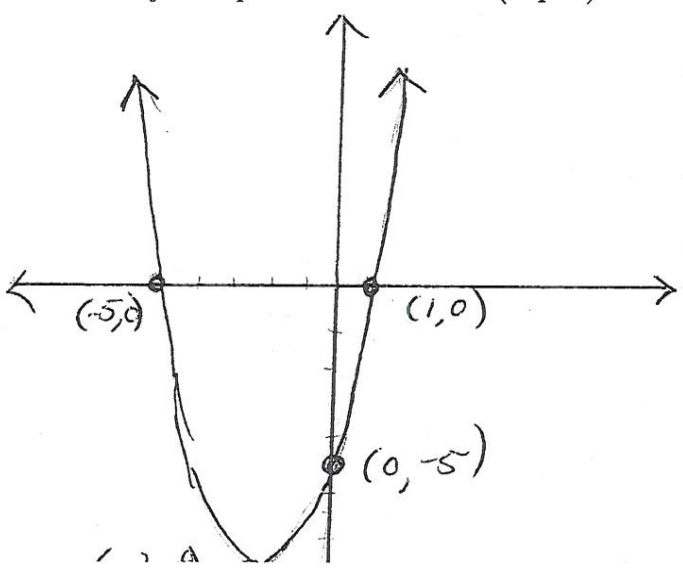
or $x = -5, 1$

$y = 0^2 + 4(0) - 5$

Answer for y-intercept: $(0, -5)$ **(1 pt)**

or $y = -5$

d). Sketch the graph of the parabola on the x-y-axis provided above. **(2 pts)**



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SECTION B – (75 PTS): In this section, you must show all algebraic work in order to obtain full credit. Partial credit will be awarded only if there is sufficient algebraic work that can lead to the correct solution. Please circle or box your answers.

B1. Use the quadratic formula to solve the equation $x^2 + 13 = 6x$ over the set of complex numbers. Write the solutions in the simplest form of $a + bi$. **(4 pts)**

$x^2+13=6x$

$x^2-6x+13=0$

$a=1$

$b=-6$

$c=13$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36-4(1)(13)}}{2}$$

$$= \frac{6 \pm \sqrt{36-52}}{2}$$

$$= \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{6 \pm 4i}{2}$$

$$x = 3 \pm 2i$$

B2. Solve $x^2 - 22x = 11$ by completing the square. **(4 pts)**

$x^2-22x+121 = 11+121$

$(x-11)^2 = 132$

$x-11 = \pm\sqrt{132}$

$x-11 = \pm 2\sqrt{33}$

$x = 11 \pm 2\sqrt{33}$

$\sqrt{132}$

$\sqrt{4}\cdot\sqrt{33}$

$2\sqrt{33}$

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B3. Solve the absolute value equation $10 - |2x - 1| = 4$ for x. **(4 pts)**

$10 - |2x-1| = 4$

$-|2x-1| = -6$

$|2x-1| = 6$

$2x-1 = 6$ or $2x-1 = -6$

$2x = 7$ | $2x = -5$

$x = \frac{7}{2}$ | $x = -\frac{5}{2}$

B4. Solve the system $\begin{cases} 5x - 7y = -16 \\ 2x + 8y = 26 \end{cases}$ **(4 pts)**

2, (−5)

$$+\begin{cases} 10x - 14y = -32 \\ -10x - 40y = -130 \end{cases}$$

$-54y = -162$

$y = 3$

$5x - 7(3) = -16$

$5x - 21 = -16$

$5x = 5$

$x = 1$

$(1, 3)$

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B5. Solve the rational equation $\frac{4}{x-3}+\frac{2x}{x^2-9}=\frac{1}{x+3}$ (4 pts)

$x \neq 3, -3$

LCD = $(x+3)(x-3)$

$4(x+3)+2x=1(x-3)$

$4x+12+2x=x-3$

$6x+12=x-3$

$5x+12=-3$

$5x=-15$

$x=-3$

$\emptyset$ no solution

B6. Simplify the complex fraction $\frac{\frac{x}{2}+\frac{2x}{3}}{\frac{2}{x}-\frac{x}{2}}$ completely. (4 pts)

LCD = $6x$

$$\frac{3 \quad 2}{6x\cdot\frac{x}{2}+6x\cdot\frac{2x}{3}}$$
$$\frac{6x\cdot\frac{2}{x}-{}^{3}6x\cdot\frac{x}{2}}$$

$$\frac{3x^2+4x^2}{12=3x^2}$$

$$\frac{7x^2}{12-3x^2}$$

B7. Solve the radical equation $x - 3 = \sqrt{27 - 3x}$ for x **(4 pts)**

$$(x-3)^2 = 27-3x$$
$$x^2-6x+9 = 27-3x$$
$$x^2-3x-18 = 0$$
$$(x-6)(x+3) = 0$$
$$x = 6, \cancel{-3}$$

B8. Simplify $\dfrac{x^2}{x^2-4} \cdot \dfrac{x^2-5x+6}{x^2-3x}$ completely. **(3 pts)**

$$\frac{x^2}{(x+2)(x-2)} \cdot \frac{(x-2)(x-3)}{x(x-3)}$$

$$\frac{x}{x+2}$$

B9. Simplify $\dfrac{4}{a^2-a-2} + \dfrac{3}{a^2+4a+3}$ **(4 pts)**

$$(a-2)(a+1) \quad (a+3)(a+1)$$

$$LCD = (a-2)(a+1)(a+3)$$

$$\frac{4(a+3)+3(a-2)}{(a-2)(a+1)(a+3)}$$

$$\frac{4a+12+3a-6}{LCD}$$

$$\frac{7a+6}{(a-2)(a+1)(a+3)}$$

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B10. Write $\frac{5}{3+8i}$ in the simplest form of $a+bi$ **(3 pts)**

$$\frac{5}{(3+8i)} \cdot \frac{(3-8i)}{(3-8i)} = \frac{5(3-8i)}{9-64i^2}$$

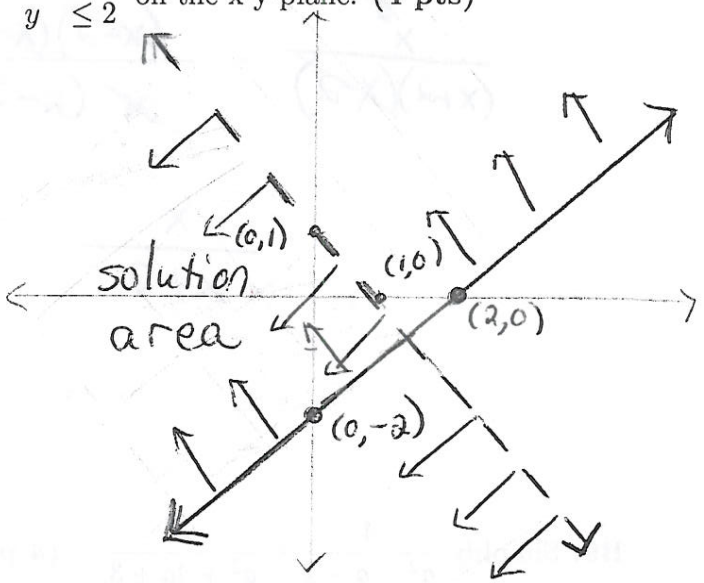
$$= \frac{5(3-8i)}{9+64}$$

$$= \frac{15-40i}{73}$$

$$= \frac{15}{73} - \frac{40}{73}i$$

B11. Solve the following inequalities.

a). Graph the solution region for $\begin{cases} x + y < 1 \\ x - y \leq 2 \end{cases}$ on the x-y-plane. **(4 pts)**



b). Graph the solution set for $x^2 - x - 2 < 0$ on a real line. **(4 pts)**

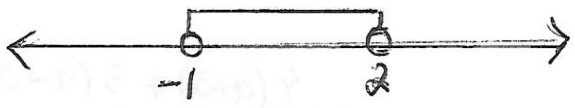
$(x-2)(x+1) < 0$

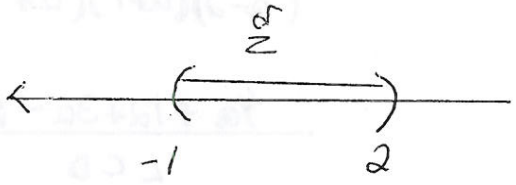
open @s: $x = 2, -1$

$x < -1$: $- \cdot -$ no

$-1 < x < 2$: $- \cdot +$ yes

$x > 2$: $+ \cdot +$ no



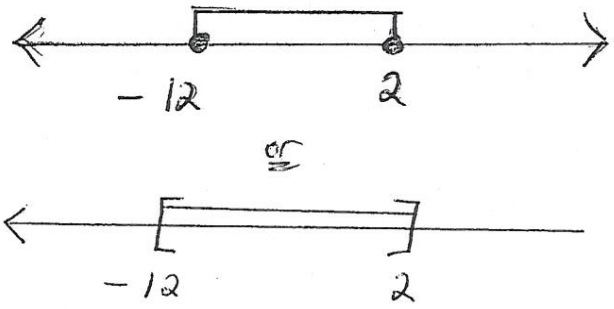


c). Solve the absolute value inequality $|m+5|+9 \leq 16$ for x. Then graph the solution set on a real line. **(4 pts)**

$|m+5| \leq 7$

$-7 \leq m+5 \leq 7$

$-12 \leq m \leq 2$



or

B12. Simplify the following expressions completely. Where appropriate, rationalize the denominators. You may assume that all variables are positive.

a). $\left(\sqrt[3]{y^4}\right) \cdot \left(\sqrt[3]{16y^5}\right)$ **(3 pts)**

$\sqrt[3]{16y^9} = \sqrt[3]{8y^9} \cdot \sqrt[3]{2} = 2y^3\sqrt[3]{2}$

b). $\dfrac{9}{6-\sqrt{10}}$ **(3 pts)** $\cdot \dfrac{(6+\sqrt{10})}{(6+\sqrt{10})} = \dfrac{9(6+\sqrt{10})}{36-10} = \dfrac{9(6+\sqrt{10})}{26}$

or

$\dfrac{54+9\sqrt{10}}{26}$

c). $\left(\sqrt[3]{x^2}\right)\left(\sqrt[6]{x^5}\right)$ Express final answer in <u>simplest radical form</u>. **(3 pts)**

$x^{2/3} \cdot x^{5/6} = x^{2/3+5/6} = x^{\frac{4}{6}+\frac{5}{6}} = x^{9/6} = x^{3/2} = x \cdot x^{1/2}$

$= x\sqrt{x}$

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B13. Solve 4 out 5 of the following word problems using an equation or a system of equations. Identify all variables and show all algebraic work. No credit will be given for solving these problems by trial and error or strictly arithmetic methods. Be sure to put a large X through the problem which you do not want to be counted. If you do not cross out any of the problems, then the last problem will be eliminated.

I). Soybean meal is 14% protein and cornmeal is 7% protein. How many pounds of each should be mixed together in order to create a 280-lb mixture that is 11% protein? **(4 pts)**

x = lbs soy
y = lbs. corn

$$\begin{cases} x + y = 280 \\ .14x + .07y = .11(280) \end{cases} \quad \times -7,\ \times 100$$

$$+ \begin{cases} 14x + 7y = 3080 \\ -7x - 7y = -1960 \end{cases}$$

$$7x = 1120$$

$$x = 160$$

160 lbs. soybean meal

120 lbs. cornmeal

II). Darren paddled for 4 hrs with a 6 km/h current to reach a campsite. The return trip against the same current took 10 hrs. Find the speed of Darren's canoe in still water. (4 pts)

r · t = d

	r	t	d
w/ current	x + 6	4	4(x+6)
against current	x − 6	10	10(x−6)

x = speed in still water

$$4(x+6) = 10(x-6)$$
$$4x + 24 = 10x - 60$$
$$84 = 6x$$
$$14 = x$$

km/hour

4

III). Zoe can rake her yard in 4 hrs. Steffi does the same job in 3 hrs. How long would it take the two of them, working together, to rake the yard? **(4 pts)**

	time	work in 1 hr
Zoe	4	$\frac{1}{4}$
Steffi	3	$\frac{1}{3}$
both	x	$\frac{1}{x}$

$$\frac{1}{4} + \frac{1}{3} = \frac{1}{x}$$

LCD $= 12x$

$$3x + 4x = 12$$

$$7x = 12$$

$$x = 1\frac{5}{7} \text{ hrs.}$$

4

IV). The St. Marks Community Barbecue served 250 dinners. A child's plate cost \$3.50 and an adult's plate cost \$7.00. A total of \$1347.50 was collected. How many of each type of plate was served? (4 pts)

c = # children
a = # adults

$$\begin{cases} c + a = 250 \\ 3.50c + 7a = 1347.50 \end{cases} \quad -7$$

$$\begin{array}{r} 1750.0 \\ -\ 1347.5 \\ \hline 402.5 \end{array}$$

$$\begin{cases} -7c - 7a = -1750 \\ 3.5c + 7a = 1347.5 \end{cases} \quad +$$

$$-3.5c = -402.50$$

$$c = 115$$

115 child's plates
135 adult's plates

$$\begin{array}{r} 115. \\ 3.5_{\wedge}\overline{)402.5_{\wedge}} \\ 35 \\ \hline 52 \\ 35 \\ \hline 175 \\ 175 \\ \hline \end{array}$$

Please Turn Over: $\Longrightarrow$

V). The pitch P of a musical tone varies inversely as its wavelength W. One tone has a pitch of 330 vibrations per second and a wavelength of 3.2 ft.

a). Find the variation constant K. **(1 pts)**

$$P = \frac{k}{W}$$

$$330 = \frac{k}{3.2}$$

$$1056 = k$$

330
3.2
660
996
1056.0

b). State the Equation of Variation. **(1 pts)**

$$P = \frac{1056}{W}$$

c). Find the wavelength of another tone that has a pitch of 550 vibrations per second. **(2 pts)**

$$550 = \frac{1056}{W}$$

$$550W = 1056$$

$$W = \frac{1056}{550}$$

$$W = 1.92 \text{ ft}$$

1.92
550 | 1056.0
550
5060
-4950
1100

Please Turn Over: $\Longrightarrow$

4